STRESS RESULTANTS OF CURVED STRUCTURE FOR TYPICAL

LOADINGS BY ISOPARAMETRIC BEAM ELEMENT

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ABSTRACT

The paper presents the analysis of curved structure as a helical staircase, supported between the two floors of the building by using finite element method. An isoparametric beam element is adopted for descratisation. Isoparametric beam element is capable of modeling the structure which is not only curved in the plan but also in elevation too. The stress resultants are predicted along the centerline length of the selected helical staircase with 270* helix angle, 3.6 m floor height and 150 mm waist-slab thickness. This structure is discretized with 15 curved elements, having 31 nodal points and 171 degrees of freedom. A curved helical staircase is analyzed for restrained boundary condition with three types of loadings. Categorically the loads selected are U. D. L over an entire span, U. D. L over the bottom half length and U. D. L over top half length of the structure. The salient values of stress resultants and displacements are shown graphically as well as in a tabular form

KEYWORDS: Curved Beam Element, Degree of Freedom, Finite Element Method, Helix Angle, Isoparametric Element & Stress Resultant

Original Article

1 INTRODUCTION

For any structure, it is an essential requirement to know an exact stress distribution under different boundary conditions and variable loading patterns. For simple structure, the stress distribution behavior can be easily found out by well known analytical methods. But an analysis of curved structure for different loading pattern and variable boundary conditions, creates complication due to the intersection of in-plane and bending stresses. Numerical methods have remarkably gained an importance for analyzing the curved/complicated structure with great accuracy and with minimum time consumption. The curved structure is analyzed by using a finite element method with isoperimetric curved beam element. The basic nodal configuration for the curved beam with 21degree of freedom is shown in figure 1.
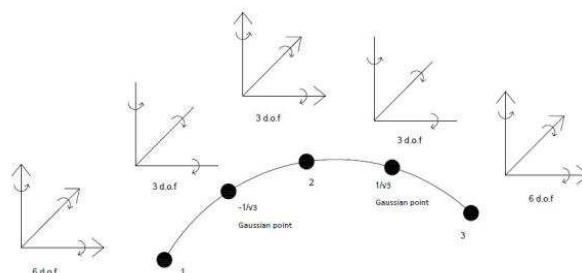

Figure 1: Nodal Configuration for Curved Beam Element

A staircase is an essential element for vertical circulation. Helical staircase occupies minimum space and providing all the architectural aspects and functions to connect the vertical floors. A helical staircase is not only curved in the plan but also curved in elevation. Due to curvilinearity in plan and elevation, the tangent to the curve, goes on changing its direction at every section.

Using the computer program, the helical staircase is selected to protect the stress resultants for restrained boundary condition and different loading patterns.

Selected helical staircase for predicting the stress resultants, has a 270* helix angle; 3.6 m floor height and 150 mm waist slab thickness with an inner radius as 500 mm and outer radius as 1500 mm. Three types of loadings are considered to study the behavior pattern of stress resultants. Uniformly distributed load of 5 kN/m over an entire span, for bottom half span length and top span length is considered for analyzing this highly curved structure.

All the results obtained for above-mentioned loads for displacements, force resultants, and moment resultants are shown graphically as well as in a tabular form.

2. METHODOLOGY

An analysis of helical staircase was carried out by a computer program for finite element method taking isoperimetric beam element for restrained boundary conditions. Rotations at loof nodes selected at $\varepsilon = +-1/\sqrt{3}$ are consistent with shell elements. Gaussian integration points are in local directions while all other variables are in global directions. There are in all 21 degrees of freedom in this element. The 21 degrees of freedom is selected to define axial forces, transverse shears, bending and torsional behavior of curved beams.

Three cases of loading were studied to analyze the helical staircase. The stress resultants thus obtained for following loading conditions.

- U. D. L over entire span length: 5 kN/m.

- U. D. L over bottom half span length

- U. D. L over top half span length Material and properties, selected for analysis are as follows

  Angle of helix: 270*

  Height of floors 3.6 m

  waist slab thickness: 150 mm

  Top and bottom supports restrained

  Area of cross section: 15 x 104 mm2

  Modulus of elasticity (E): 0.12 x 105 N/mm2

  Shear modulus (G): 0.52173 x 104 N/mm2

  Moment of inertia (Izz):1.25 x 108 mm4

  Moment of inertia (Iyy):2.8125 x 108 mm4

  Moment of inertia (Ip):10.125 x 108 mm4

Unit weight of concrete (e): 24KN/m3

Poisson's ratio of concrete(r): 0.15

3. OBSERVATION

The computer program was run and the results were obtained to predict the displacements; shear and moments at every nodal point for all the three loading cases. The results thus obtained are shown graphically and the salient valves are shown in a tabular form.

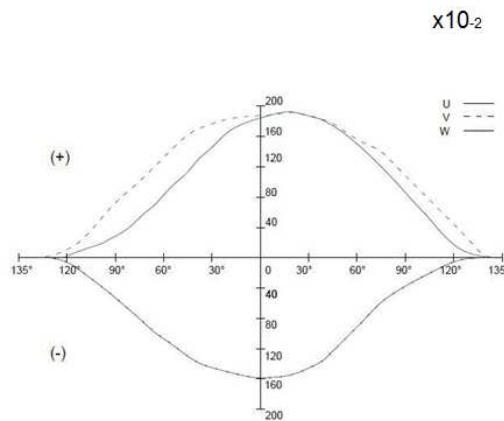All the displacements in u, v and w directions for the uniformly distributed load of 5 KN/m over an entire span are shown graphically in figure 2.

Figure 2: Displacement for U. D. L throughout in mm

All the displacements in u, v and w directions for uniformly distributed load over the bottom half span length is shown graphically in figure 3.
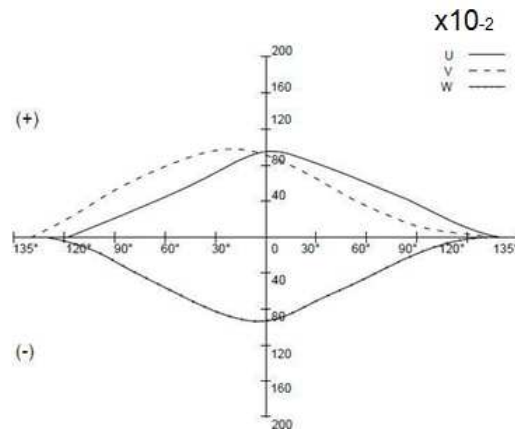
Figure 3: Displacement for U. D. L Bottom Half in mm

All the displacements in u, v and w directions for uniformly distributed load over bottom top span length is shown graphically in figure 4
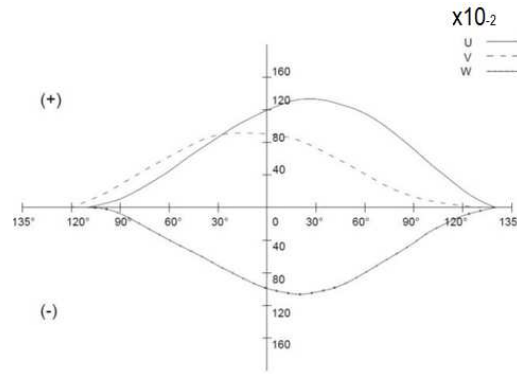
Figure 4: Displacement for U. D. L Top Half in mm

Tension is shown by +ve sign of compression by - ve sign

All the force resultants as Fx for all the three types of loadings are shown graphically in figure 5
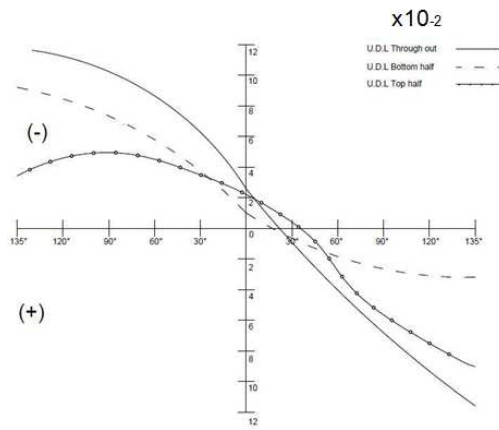

Figure 5: Fx: For all Three Loadings in Kn

force resultants as Fxy for all the three types of loadings are shown graphically in figure 6.
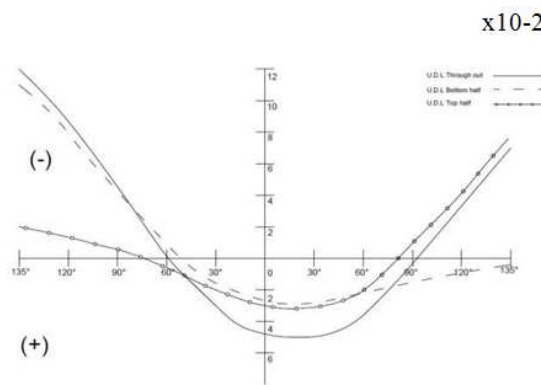

Figure 6: Fxy: For all Three Loadings in Kn

force resultants as Fxz are shown by a graph for all the three loadings in figure 7
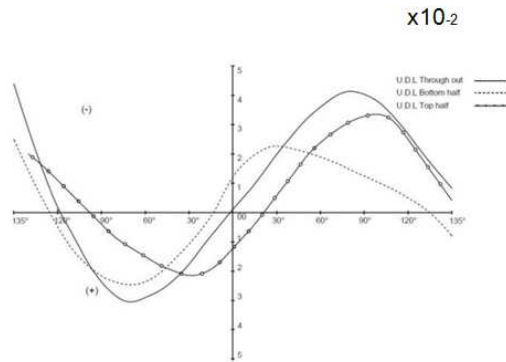
x10⁻²

Figure 7: Fxz: For all Three Loadings in Kn

3.1 Moment Resultants

For all the three loads (U. D. L throughout, U. D. L at bottom half-length, U. D. L at top half length) are applied to the helical staircase to predict the moment resultants in all the three directions.

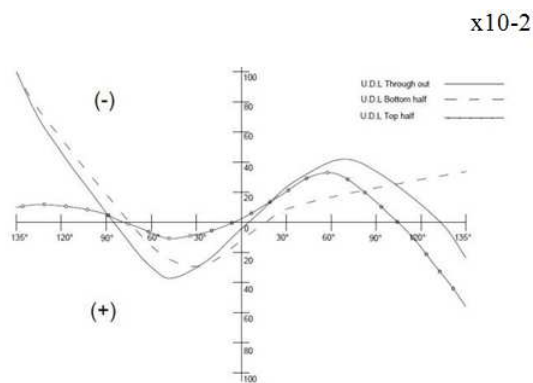Moment resultants Mxy is shown graphically in figure 8 for all the three types of loads, mentioned above

x10-2

Figure 8: Mxy: For all Three Loading in Knm

Moment resultants Mxz is shown graphically in figure 9 for all the three types of loads.
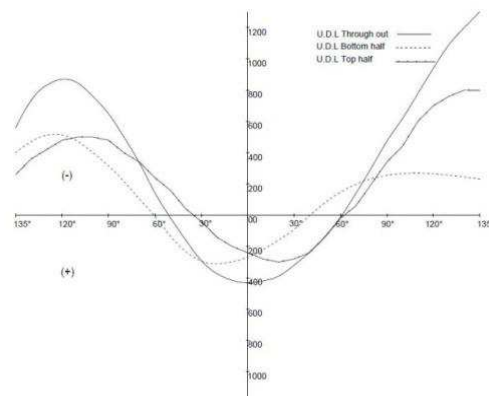

Figure 9: Mxz: For all Loadings in Knm

Torsional moments "TOR" are shown graphically for all the three loadings, mentioned above, in figure 10
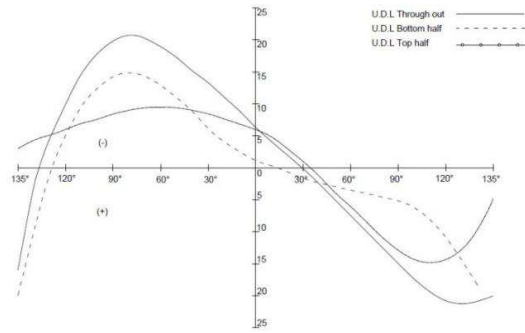
Figure 10: Tor: For all the Loadings in Knm

salient values for nodal displacements for all the three types of loading are shown in table no 1

Table 1: Shows Nodal Displacements in MM

Displacement	U. D. L through Out Span	U. D. L Bottom Half Span	U. D. L Top Half Span
U	1.92 mm	0.89 mm	1.32 mm
V	1.7 mm	0.98 mm	0.94 mm
W	1.61 mm	0.87 mm	1.04 mm

salient values of force resultants for uniformly distributed load over an entire span, uniformly distributed load for bottom half span and for top half span length of the helical staircase are shown in table no 2.

Table 2: Force Resultant in KN

Force Resultant	U. D. L Through Out Span	U. D. L Bottom Half Span	U. D. L Top Half Span
Fx	11.65 KN	8.9 KN	9.42 KN
Fxy	12.04 KN	10.85 KN	7.63 KN
Fxz	4.45 KN	2.53 KN	3.33 KN

salient values of moment resultants for uniformly distributed load over an entire span, uniformly distributed load for bottom half span and uniformly distributed load for top half span length of the helical staircase are shown in table no 2.

Table 3: Moment Resultant in KNm

Force Resultant	U. D. L Through Out Span	U. D. L Bottom Half Span	U. D. L Top Half Span
Mx	10 KNm	9.6 KNm	5.44 KNm
Mxy	6.4 KNm	2.4 KNm	4.95 KNm
Tor	2.13 KNm	2.06 KNm	1.5 KNm

4 CONCLUSIONS

It is concluded that the effect of vertical shear and torsion is less important as compared with transverse shear and moment.

The stress resultants show the sinusoidal behavior, under different loading conditions.

The torsional effects are negligible in the helical staircase.

For all types of loading, selected in this paper, the 'u' displacement is maximum near the center of the helix.

The 'v' displacement behaves like a sinusoidal wave and the maximum values occur near 1/3 rd distances from the center of the helix.

The maximum values for 'W' displacement take place near center of the span for all types of loadings.

For all the cases, mentioned above, the deflections are always within the permissible limits for the selected thickness of the staircase.

Although it is assumed that helical staircase, being highly curved structure, requires more than adequate section and reinforcement and especially to resist the torsional moments as compared with traditional staircase. But it is concluded and proved from this available analysis, that if both the supports are firmly restrained, no additional depth and reinforcement is required to be provided for such a highly curved structure. Even nominal torsional reinforcement is adequate to safeguard the structure from the torsional moment.

ACKNOWLEDGEMENT

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